I

Algebra and Broken Bones

Charlie is doing things his own way. He believes in self-expression, and he doesn't understand why Mr Barton is always trying to change the way he approaches things. Take algebra, for example. Algebra is just another word for finding the missing number. Charlie has sat and listened to Mr Barton's explanation about 'what you do to one side, you have to do to the other', but it all sounds a bit communist for his liking, and he has always been able to solve problems by his own special methods.

So, after disturbing the boy in front of him for several enjoyable minutes by gently kicking the bottom of his chair, he settles himself down to deal with the problems Mr Barton is writing on the board with a general feeling of confidence. $2x + 11 = 21$. No worries – the $2x$ must be the same as 10 because $10 + 11 = 21$, and so x must be equal to 5. Charlie gives himself a tick, because he is pretty sure that Mr Barton will disagree with how he has gone about this problem. He moves on to the next question, $4y - 9 = 19$. Once again, Charlie's method works flawlessly: $y = 7$. He celebrates by stealing the rubber off the desk next to him. Next, $3t - 5 = 2t + 1$. Oh dear…

Charlie feels a dark, sinking feeling deep inside him. Nothing is certain any more. How does he deal with the madness that he is suddenly faced with? There are two 't's in this problem. What do you do when there are two 't's? He whispers his question to his next-door neighbours, but he has already alienated them by kicking their chair and stealing their rubber respectively. He surrenders completely to a feeling of total helplessness, and lets his head slowly sink into his hands. Once again, the dark forces of mathematics have combined to defeat him. It won't be long before Mr

Barton senses this, and comes to mock him in his defenceless state. He can hear the squeaking of dirty brown brogues now.

At school level, algebra is the study of generalized number. It can range from being incredibly abstract to reasonably concrete. If you write down the symbol 'x' and nothing else, that symbol stands for 'any number at all'. It is as if, were you to look underneath the symbol, you would find a box that contains all of the infinite numbers in existence. Slightly less abstract, but definitely not specific, is an expression like $2x$. In this case, $2x$ stands for 'two times any of the numbers that exist', which, in fact, means that it also stands for any number at all, since every number is twice another number. Hmmm…

Things become a little clearer, when you start attaching such expressions to particular situations. A middle-aged woman refuses to tell you her age, and so you can refer to her age as being x (although in this case, the circumstances of the situation limit the possibilities for x – she can't be a negative age, and she can't be older than 150). She also informs you that her son is thirty years younger than her. This doesn't help you to work out the age of her son, but it does mean that you can refer to his age as being $(x - 30)$. Finally, she lets slip that the number on a passing bus is three times the age she will be in five years time. Again, this is no help at all if you want to know the number on the bus, but it does allow you to express it in terms of x. In five years time, the woman will be $(x + 5)$ years old. The number on the bus is three times this, and so it can be expressed as $[3 \times (x + 5)]$.

All of these are examples of algebraic expressions. In each case, the unknown (or variable) potentially stands for all the numbers in existence. If you want to get more particular, you need more information. If you get more information, you might be able to form some equations. To form an equation you need to know that two different things have the same value. For example, if the lady admits that her son is 20, you know that $(x - 30)$ is equivalent to 20, and you can write down the equation $x - 30 = 20$. This equation has only one solution, 50.

Not all equations are as specific. It is possible to come up with equations that have as many solutions as you like. For example, $x^2 = 16$ has two solutions (4 and -4), $(x-1)(x-2)(x-3) = 0$ has three solutions (1, 2 and 3), and $x = x$ has an infinite number of solutions.

> **53. The swimming pool at the hotel in Akagera Game Park needs to be topped up, but the plumbing has once again been destroyed by a rogue elephant. The staff are able to work together to fill it at a rate of 20 litres per minute. If there were 2000 litres of water in the pool at the start of the operation, write an expression for the amount of water after m minutes.**

So far, I have mentioned expressions and equations that only contain one unknown, but it is perfectly possible to come up with equations and expressions that contain two or more unknowns. The lady can tell you that the number of ice creams that she has eaten in the course of her life is equal to the sum of her age and her husband's age. Then if you call her age x, and her husband's age y, the number of ice creams that she has eaten is the expression $x + y$. If she goes on to say that the number of ice creams she has eaten is 80, you can write down the equation $x + y = 80$. This equation is called indeterminate because there are an infinite number of possibilities for x and y. The lady could be 40 and her husband could be 40, or they could be 41 and 39, or 100 and -20, or 79½ and ½, although some of these are physically impossible and some of them are illegal.

In order to fix the values for x and y, you need more information. For example, if she also tells you that she is older than her husband, and their difference in age is 20, you can write down the equation $x - y = 20$. You now have two equations that must be true at the same time. Such equations are called simultaneous equations. In this case, you now have enough information to find the value of x and y, but this is not always true of simultaneous equations. They can still be indeterminate. In general, if you have two variables, you will need at least two equations connecting them, if

you have three variables, you will need at least three equations connecting them, and so on, and so on.

Once you have come up with the idea of representing an unknown number by a symbol, the next difficulty that faces you is how to deal with them. The history of elementary algebra consists of the various inventions and improvements that different cultures have come up with to deal with algebraic expressions and equations.

Some techniques have been discarded, as better ones have been found, and there are a couple of examples of these in the chapters to come. The techniques that have survived form the basis of what you were taught at school. I am sure you will remember instructions like 'do the same to both sides', and 'cross-multiply', and 'expand the bracket by multiplying everything inside it'. It is these 'laws' of algebra that are the problem. They form a series of logical steps which lead you towards a solution of an equation or a simplification of an expression – but it is often not at all clear where the logic lies. They often feel like a series of commands from an unsympathetic despot, whose one aim is to force you to fill pages of paper with strange symbols according to random rules. But I hope that in what follows, it will become clear that all the rules of algebra, both those that have fallen out of use and those that Mr Barton continues to drill into his students, have their roots in an understanding of how numbers in general work.

The notion of pain is embedded in the word 'algebra' itself. 'Algebra' is an 'Englishification' of the Arab word '*al-jabr*', which al-Khwarizmi used as a technical term in connection with solving equations. It describes the process of taking a term from one side of an equation to the other. For example, if you have the equation $4x = 2 - 2x$, you can 'take the $2x$ to the other side', and write $4x + 2x = 2$. However, *al-jabr* can also describe the process of setting a broken bone. There are probably many people in the world who feel that the two operations are equally gruesome.

In Medieval Spain, due to the influence of the Moors, a barber called himself an *algebrista*, because he generally did a bit of bone-setting and blood-letting on the side to supplement the income he received from styling hair. That is why the traditional sign for a barber is a red-and-white striped pole – the red and white symbolize blood and bones.

Rumour has it that Stephen Hawking was instructed by his publishers not to include an equation anywhere in the first half of a scientific book that he intended for the general public. It was apparently felt that the sight of an equation would immediately cause the average reader to put the book back on the shelf and walk hurriedly away.

54. Think of a number. Add four, and double your answer. Then subtract eight, and divide by two. Your answer will always be the number you started with. How does this trick work?